

Rapid non-adiabatic loading in an optical lattice

Xinxing Liu, Xiaoji Zhou,* Wei Xiong, Thibault Vogt, and Xuzong Chen

*School of Electronics Engineering and Computer Science,
Peking University, Beijing 100871, People's Republic of China*

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We present a scheme for non-adiabatically loading a Bose-Einstein condensate into the ground state of a one dimensional optical lattice within a few tens of microseconds typically, i.e. in less than half the Talbot period. This technique of coherent control is based on sequences of pulsed perturbations and experimental results demonstrate its feasibility and effectiveness. As the loading process is much shorter than the traditional adiabatic loading timescale, this method may find many applications.

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Numerous works related to optical lattice trapping have been published, especially using coherent atoms or Bose-Einstein condensate (BEC) [3], since it has various applications in quantum computation, simulation of basic condensed matter physics, atomic clocks, *etc.* A common concern to those experiments is how to load a BEC into the lattice without excitation or heating [4–6]. In most cases, one chooses to turn on the light field adiabatically and the loading process usually lasts up to tens of milliseconds, during which one tries to avoid excitations to higher states. A long loading may be problematic for quantum computation experiments as it increases the time during which decoherence can occur and reduces speed when atoms stored in optical lattices shall be interrogated and reloaded several times. Furthermore, adiabatic loading is difficult to obtain near the critical point of phase transitions for finite temperatures even well below the BEC critical temperature.

An alternative idea is to prepare the BEC in the ground state of the lattice, and then turn on the lattice suddenly. This 'preparing' process can be much shorter compared to the adiabatic loading (about 30 μs , as we can see later in this article). One way for realizing this 'preparing' process can stem from nonholonomic coherent control [1, 2, 7]. In this approach, a sequence of two well chosen Hamiltonians are imposed on the system and the preset duration of each step is modified according to a defined cost function, in order to get the aimed evolution operator as well as the target state.

Our proposal for designing and computing the preloading process is reminiscent of this method, but we focus more on the feasibility of its experimental implementation, rather than the stringency or universality of the approach. Experimentally, we achieve the non-adiabatic loading with a very few pulses using directly the results of our computational design. According to our analysis, this method would be valid on many other occasions, such as

loading the condensate directly into excited states of the lattice. We first introduce a general method for determining the sequence of steps to be applied to the system before giving an account of our experimental results.

Suppose that before the sudden loading at time t_0 of the lattice with optical depth V_0 , m steps have been applied. The i th step corresponds to a Hamiltonian \hat{H}_i kept constant during t_i . The final state $|\psi(t_0)\rangle$ is given by:

$$|\psi(t_0)\rangle = \prod_{i=m}^1 \hat{U}_i |\psi_0\rangle \quad (1)$$

where $\hat{U}_i = e^{-i\hat{H}_i t_i/\hbar}$ is the evolution operator of the i th process. With $|\psi_a\rangle$ the aimed state, which is the ground state of the lattice in the context of this paper, the total population of the excited states N_e at time t_0 , is:

$$N_e = 1 - |\langle\psi_a|\psi(t_0)\rangle|^2 \quad (2)$$

Our goal is to properly choose \hat{H}_i and t_i so that N_e is small enough to be neglected in the experiment, or the final state $|\psi(t_0)\rangle$ to be close enough to $|\psi_a\rangle$.

The target state $|\psi_a\rangle$ can be obtained simply if atomic interactions are neglected. The effect of the optical lattice on the atoms corresponds to a stationary periodic potential with periodicity $\lambda_L/2$, spontaneous emission being neglected since the laser used for the lattice is sufficiently far detuned. So neglecting atomic interactions, the Hamiltonian of the interaction of atoms with the optical lattice is:

$$\hat{H}_0 = \frac{\hat{p}^2}{2m} + \frac{V_0}{2}(1 + \cos 2k_L x) \quad (3)$$

where m is the atom mass, k_L is the wave number of lattice light and V_0 the lattice depth and \hat{p} is the momentum of each atom. The corresponding Talbot time [8] is $T_T = \frac{2\pi}{4\omega_R}$, where $\omega_R = \frac{\hbar k_L^2}{2m}$ is the one photon recoil frequency. Because of the periodicity of the potential, $|\psi_a\rangle$ can be decomposed over a reduced basis of plane waves

*Electronic address: xjzhou@pku.edu.cn

$|2n\hbar k_L\rangle$. One obvious choice for each \hat{H}_i is to take the Hamiltonian corresponding to the interaction of atoms with a standing wave with same periodicity $\lambda_L/2$ so that the Hilbert space is limited to the basis of states $|2n\hbar k_L\rangle$. For this purpose, the power of the same laser as the one used for the final lattice loading is simply adjusted and each Hamiltonian \hat{H}_i is obtained from Eq. (3) after substitution of V_0 by the new lattice depth V_i .

More precisely, as \hat{H}_i has spatial periodicity, we get its eigenstates by solving the equation $\hat{H}_i|n, q\rangle = E_{n,q}|n, q\rangle$ [9], where $|n, q\rangle$ is a Bloch state with n the band index and q the quasi momentum and $E_{n,q}$ is the corresponding eigenenergy. We use the notation $|n, q, V_i\rangle$ for denoting the Bloch states for a V_i lattice depth. Since only Bloch State with $q = 0$ is populated initially, no other quasimomenta can be populated during the sequence of pulses. Thus the state of the system can be spanned over the momentum eigenstates basis $|2\ell\hbar k_L\rangle$, independent on the potential depth V_i and the evolution operator can be written as the following matrix: $\mathbf{U}_i(V_i, t_i) = \mathbf{C}(V_i)\mathbf{E}(V_i, t_i)\mathbf{C}(V_i)^\dagger$, where $\mathbf{C}(V_i)$ is the unitary matrix of transition between the Bloch States basis and the momentum eigenstates basis with matrix elements $\mathbf{C}(V_i)_{\ell n} = \langle 2\ell\hbar k_L|n, q=0, V_i\rangle$ and where $\mathbf{E}(V_i, t_i)$ is a diagonal matrix with elements $\mathbf{E}(V_i, t_i)_{nn} = \exp(-iE_{n,q=0}(V_i)t_i/\hbar)$. Because of the simple form of the potential, these matrices are easily obtained and from those the wave function's evolution is solved numerically.

A traditional simple approximation to solve this problem of a standing wave of light pulse interaction with atoms is to omit the effect of kinetic energy. As shown in the works [8, 10], the motional term in the Hamiltonian may be neglected. For a square pulse, when the depth of the standing wave potential is V and the duration of the pulse is t , the wave function after the pulse can be expressed as $\psi(t) = \psi(0)e^{-i\frac{V-t}{2\hbar}\cos 2k_L x}$ and decomposed over different momentum states $|2n\hbar k_L\rangle$, using the Bessel functions. This approximation is valid only in the Raman-Nath regime, when the pulse is short enough that the displacement of the atoms is much smaller than the lattice period. From references [11, 12], we can see experimental data deviate distinctly from the Bessel functions when the pulse duration is longer. For multi pulse cases, the motion term may be neglected during each pulse but taken into account during the intervals between pulses. This approximation is valid only when the sum of all the pulses' durations is far less than the Talbot time T_T . In [8], $T_T = 10 \mu\text{s}$, and two 100 ns pulses were applied, while the interval time can be up to $10 \mu\text{s}$. However, as the total pulse duration 200 ns is far less than T_T , the whole process is still in Raman-Nath regime and we can see the theoretical prediction agrees well with the experimental results.

We compare in Fig. 1 the results of the two methods with our experimental data for the case of a single square pulse applied to the system. In our experiment, we first prepare a BEC of about 2×10^5 ^{87}Rb atoms

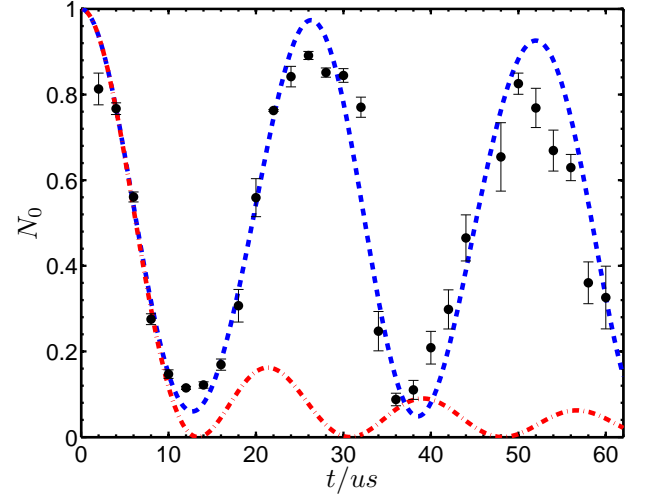


FIG. 1: (color online) Relative atom number N_0 in $p = 0\hbar k_L$ mode versus the pulse duration t of a single standing-wave pulse applied to the system. The black points with error bars are experimental data and the red dashed-dotted line is calculated with the simple method where the kinetic energy is neglected while the blue dashed line is given by the general method (see text).

in the $|F = 2, m_F = 2\rangle$ hyperfine ground state in the magnetic trap, with longitudinal length $L = 100 \mu\text{m}$ and width $l = 10 \mu\text{m}$ [13]. The BEC is then loaded into a one-dimensional optical lattice with light wavelength $\lambda_L = 852 \text{ nm}$, along its axial direction. We suddenly turn on the standing-wave potential to a specific depth, hold the lattice for different durations t , release the condensate and measure the relative atom number N_0 in $p = 0\hbar k_L$ mode. Without considering the atomic motion, the Bessel function (red dashed-dotted line) failed to fit the experiment data when t is larger than about $10 \mu\text{s}$. On the other hand, the numerical solution taking the motional term into account (blue dashed line) predicts the experiment accurately up to $60 \mu\text{s}$, although some other effects have been neglected, such as interaction between atoms and non-zero momentum width of the condensate. As a result, we will use the general method as it will impose less restriction on our sequence design. Note that $T_T \approx 79 \mu\text{s}$ in our configuration and the fitted potential depth is $18E_R$, where $E_R = (\hbar k_L)^2/2m$ is the one photon recoil energy.

In a first attempt to design the time sequence, we use four steps, each corresponding to the application of a potential with depth V_i and a duration t_i as shown in Fig. 2a. Thus, from these eight free parameters V_i, t_i with $i = 1$ to 4 , we can obtain the evolution operators \mathbf{U}_i and the exciting rate N_e according to our previous analysis. If we want to get the minimum of N_e , it requires to test all the combinations of the eight free parameters and need too much computing time. However, we find convergence to a local minimum is easy to obtain. For this

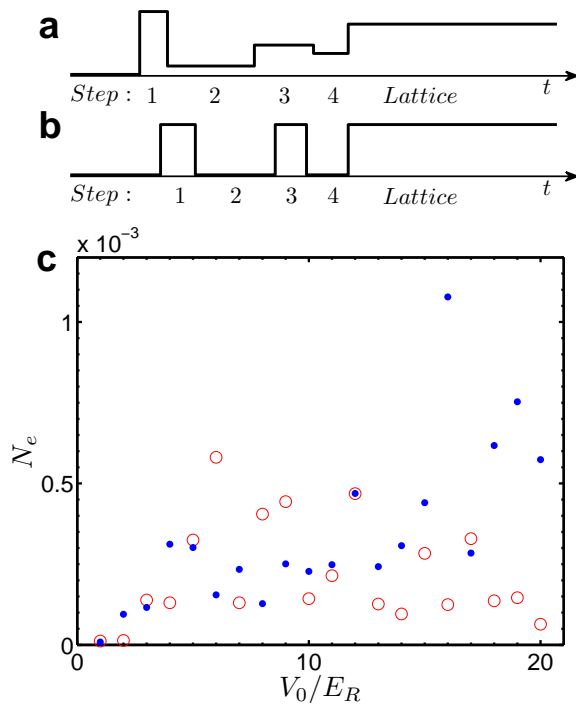


FIG. 2: (color online) Two schemes for the non-adiabatic loading. (a) Four steps pre-loading sequence with both the potential depth and the duration of each step set as free parameters. (b) Four steps pre-loading sequence with fixed potential depth and variable duration in each step. (c) Corresponding relative population N_e in excited Bloch states as a function of lattice depth, with red circles for scheme (a) and blue points for scheme (b).

purpose we modify the parameters step by step, with a small change of one parameter every step, in order to get a smaller N_e . If no smaller result for N_e can be obtained by changing any of these parameters, the minimization of N_e is stopped and the corresponding combination of parameters is the result of our design. Note that different initial values of the parameters may result in different local minima. Actually we try more than one group of initial parameters in order to get a better result, though the optimized pulse sequences obtained for different and often lower lattice depths are already good input values. As shown in Fig. 2c, the exciting rate N_e remains lower than 0.1%. The whole pre-loading process lasts for about 30 μs , which is beyond the Raman-Nath Regime but still within the limit of our method.

Nevertheless, it is not easy for this scheme to be experimentally implemented. As the lattice depth varies with time, we need a feed back loop to control the laser power to get a stable optical lattice as the atoms are very sensitive to fluctuation of the laser power. We also need sharp rising and falling edges to ensure each step duration is sufficiently similar to the designed one. If 0.5 μs is set for the upper limit of each rising and falling edge, the bandwidth of the power controlling loop should be larger

TABLE I: Time duration of each step t_i (μs) for different lattice depths V_0 with unit E_R .

V_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
4	10	9.5	4	5	6	5	9	9
8	6.5	10.5	5	6	6	4.5	10.5	7
12	5	11.5	4.5	6	5.5	4	11.5	5.5
16	4.5	12	3.5	6	5	4	12.5	4.5
20	3.5	13	3.5	5.5	6	3.5	12.5	3.5

than 1 MHz, which is hard to achieve. We would rather choose the scheme illustrated in Fig. 2b, where the step V_1 and V_3 are fixed at V_0 , and the step V_2 and V_4 are fixed at 0, t_i still being free parameters. This means no feed back loop but only a switch is needed so that the rising and falling time can be decreased to 200 ns easily. In Fig. 2c, we can see the exciting rate is still lower than 0.1% for most lattice depths lower than 20 E_R , although some degrees of freedom of our sequence of pulses are locked. In TABLE I we show the designed time sequences for several lattice depths.

Furthermore, we can transfer the state $|\psi_a\rangle$ back to the initial state $|\psi_0\rangle$ after releasing the lattice, using a similar process. TABLE I gives also the designed sequence, with time durations t_5 to t_8 and corresponding potentials, V_5 and V_7 being set to 0, while V_6 and V_8 equal to V_0 (see Fig. 3b). Note that this sequence is equivalent to the timely inverted pre-loading sequence if we ignore some minor deviations.

To demonstrate the feasibility of our proposal, we did the experiment according to three different schemes. In the first scheme (Fig. 3a), we non-adiabatically load the BEC into the lattice, according to the designed process, hold it in the lattice for $t_h = 10$ ms and release the atoms. We can see the interference peaks, similar to the familiar pattern observed in adiabatic loading experiments, which indicates a successful loading without significant excitation and heating. In the second scheme (Fig. 3b), after the same loading and holding process, we use two additional pulses to transfer the atoms back to the original state $|\psi_0\rangle$. Compared with the third scheme (Fig. 3c), where we directly turn on and off the lattice light without the pre-loading or post-releasing processes to prevent excitation but hold for the same period, the second scheme has little heating or disturbing effect on our BEC, which proves the effectiveness of our 'preparing' process of the lattice ground state. A small heating may still be observed at 12 E_R . It may be due to interaction between atoms which are not included in our model.

Fig. 4 shows the evolution of the wave function under a pulsed scheme shown in Fig. 2b. We can see that atoms can be transferred between different momentum orders only when the pulse is on. In the intervals, the amplitudes of different components keep constant, while the phases vary linearly. The phase of the $\pm 4\hbar k_L$ order evolves four times faster than that of $\pm 2\hbar k_L$ as the ki-

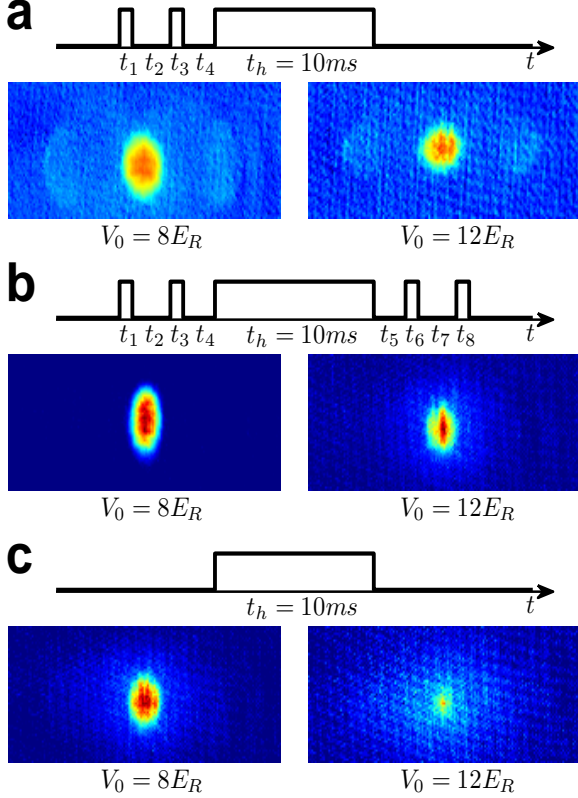


FIG. 3: (color online) Absorption images for three different time sequences. As the condensate was released from both the lattice potential and the magnetic trap, it expands freely for 30ms and then an absorption image is taken. For each time sequence, we show the results for two lattice depths of $8E_R$ and $12E_R$.

netic energy is four times larger. After switching on the optical lattice ($t = 0 \mu s$), the amplitude and phase of all components keep almost unchanged as the wave function is in the ground state of the lattice. Note that the phase jump of the non-zero momentum components at $t = -27 \mu s$ is not significant since the amplitude of these components before that time is zero.

By simply replacing the initial state $|\psi_0\rangle$ and the aimed state $|\psi_a\rangle$, our pulse sequence design is also applicable to other situations. For example, by setting $|\psi_0\rangle = |n = 0, q = 0, V_1\rangle$ and $|\psi_a\rangle = |n = 0, q = 0, V_2\rangle$, we can get a process that can change the lattice depth from V_1 to V_2 non-adiabatically without excitation. We can also load the BEC directly to symmetric excited states such as $|n = 2, q = 0\rangle$, by setting them as $|\psi_a\rangle$. Our numerical simulation shows when V_0 is around $12 E_R$, there exists theoretical possibility to load more than 99% of the

atoms to $|n = 2, q = 0\rangle$. This state can be used for studying spontaneous transition from high lying Bloch bands to lower bands experimentally. If the optical lattice is

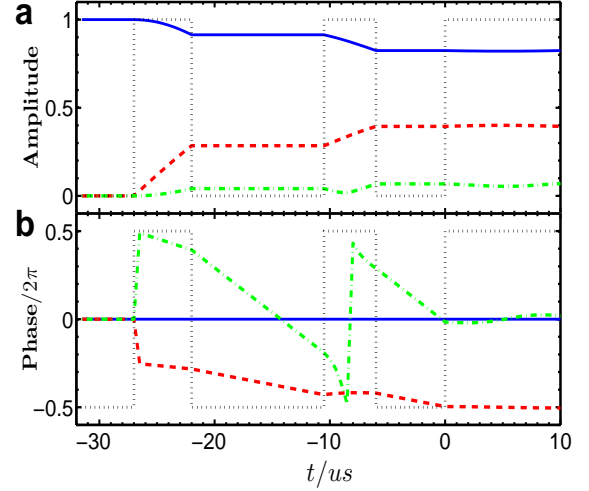


FIG. 4: (color online) Evolution of the different momentum components of the wave function. Amplitude (a) and phase (b) for the $p = 0\hbar k_L$, $\pm 2\hbar k_L$ and $\pm 4\hbar k_L$ modes are shown with blue solid lines, red dashed lines and green dashed-dotted lines respectively. The black dotted lines in both figures indicate the corresponding time sequence of the standing wave potential.

accelerated, anti-symmetric states and states with $q \neq 0$ could also be loaded.

In general, our method can be used to produce states whose momentum components are discrete and separated equally by $2\hbar k_L$. For example, we can divide one BEC into two momentum modes $|\pm 2m\hbar k_L\rangle$ equally, with negligible population in other orders. This technique is useful in atom interferometry and its theory has been developed in reference [10], while neglecting the motional term in the Hamiltonian, as discussed earlier in this article. Our method is not restricted by the Raman-Nath regime and thus gives more freedom for designing the pulse sequence.

In conclusion, we proposed a method reminiscent of the nonholonomic coherent control technique for loading the BEC into one dimensional optical lattice non-adiabatically, within a much shorter loading time (usually less than $T_T/2$) than the commonly applied adiabatic method (much longer than T_T). Our experimental results demonstrate the validity of this method. We claim that this numerical design process can be applied to various topics related to the interaction of a standing wave of light with ultra-cold gases.

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